THE TURBULENT NONISOTHERMAL FLOW OF AN INCOMPRESSIBLE GAS IN A CIRCULAR ROTATING PIPE

B. A. Kolovandin, O. A. Martynenko, and V. E. Aerov

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The nonisothermal, fully developed turbulent flow of an incompressible gas in a circular rotating cylindrical pipe is discussed. Equations are given for one-point second moments of velocity and temperature pulsations. The results of calculating the thermal characteristics are given.

In this paper we propose a method for calculating the dynamic and thermal characteristics of the completely developed turbulent flow of an incompressible gas in a circular cylindrical pipe which essentially consists in using the equations for the change in the second moments of the pulsating variables to close the averaged equations of momentum and heat transport.

A feature of this method in connection with the calculation of isothermal turbulent flows in pipes was described, for example, in [1, 2]. An investigation of the effect of the rotation of the pipe on the transition from turbulent to laminar flow was made in [3]. Here we calculate the critical Reynolds number as a function of the angular velocity of rotation of the fluid based on a method similar to that described in [1]. A method for calculating the characteristics of nonisothermal flows is developed in [4], where it was proposed to use the equation for the change in the pulsation thermal flows $\rho c_p \overline{u_i' t'}$ to calculate the fundamental thermal characteristics.

On the basis of the method described in [4] we calculate below the thermal characteristics of a completely developed turbulent flow of an incompressible gas in a circular pipe and we study the effect of the rotation of the fluid on the change in the averaged temperature profile and on the distribution of the pulsation thermal flows.

We consider the fully developed turbulent nonisothermal flow of an incompressible gas in a circular cylindrical pipe, rotating about its axis of symmetry with angular velocity ω. The origin of a cylindrical coordinate system (z, r, φ) is on the axis of symmetry, the z-axis coinciding with the direction of the fundamental motion.

Starting from the equation for the change in the Reynolds stresses in a diffusionless approximation [3], we can write the characteristics of the pulsation motion as:

$$\overline{vw} = 0, \tag{1}$$

$$\overline{uv} = -\frac{R_E^2(c_{uu} R_E + c_{1uu})}{R_L} \frac{1}{R_*^2 l^2}, \tag{2}$$

$$\overline{uw} = \frac{l^2 R_\omega}{k_u R_E + c_{1uu}} \overline{uv}, \tag{3}$$

$$\overline{v^2} = \overline{w^2} = -\overline{uv}\, \frac{k_u R_E + c_{1uu}}{R_L} \left[1 - \frac{l^4 R_\omega^2}{(k_u R_E + c_{1uu})^2} \right], \tag{4}$$

$$\overline{u^2} = 2\overline{\varepsilon} - (\overline{v^2} + \overline{\omega^2}), \tag{5}$$

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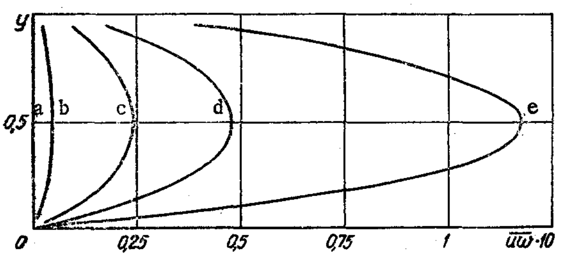


Fig. 1

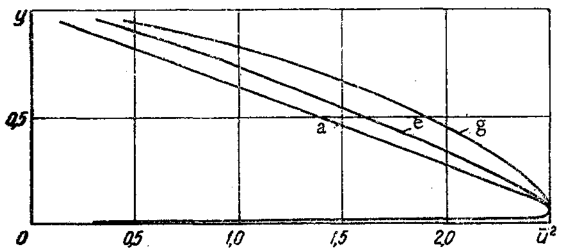


Fig. 2

Fig. 1. Distribution of $\overline{uv}$ at a section of the pipe: a) $R_\omega = 0$; b) 10^2; c) $0.5 \cdot 10^3$; d) 10^3; e) $2.5 \cdot 10^3$.

Fig. 2. Distribution of the intensity of longitudinal pulsations: a) $R_\omega = 0$; e) $2.5 \cdot 10^3$; g) $4 \cdot 10^3$.

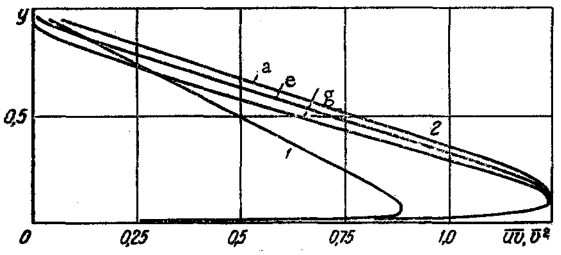


Fig. 3

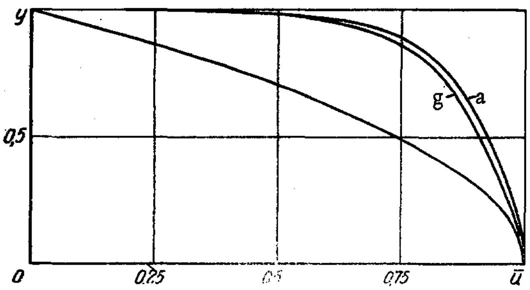


Fig. 4

Fig. 3. Distribution of the intensity of transverse velocity pulsations and tangential stresses: 1) $\overline{uv}$; 2) $\overline{v^2} = \overline{w^2}$; a) $R_\omega = 0$; e) $2.5 \cdot 10^3$; g) $4 \cdot 10^3$.

Fig. 4. Averaged velocity profile: a) $R_\omega = 0$; g) $4 \cdot 10^3$.

$$\sqrt{\overline{\varepsilon}} = \frac{R_E}{R_* l}, \tag{6}$$

where the local and turbulent Reynolds numbers are defined by the numerical solution of a system of equations for the Reynolds numbers and the energy balance in the pulsation motion, which have the form

$$R_L + \frac{R_E^2 (c_{uu} R_E + c_{1uu})}{R_L} = -l^2 R_*^2 y, \tag{7}$$

$$R_L = -(k_u R_E + c_{1uu}) \sqrt{\frac{R_E + \dfrac{c_{1uu}}{c_{uu}}}{\dfrac{2}{3}\left(\dfrac{k_u}{c_{uu}} - 1\right) R_E} \left[1 + \frac{l^4 R_\omega^2}{(k_u R_E + c_{1uu})^2}\right]}. \tag{8}$$

From the solutions of (7) and (8) we can find the distribution of the Reynolds numbers R_L and R_E for the case when the integral scale of turbulence is given in the form of a function of the transverse coordinate. Here l is the integral scale of turbulence,* and we assume that it can be approximated by a function of the y coordinate only.

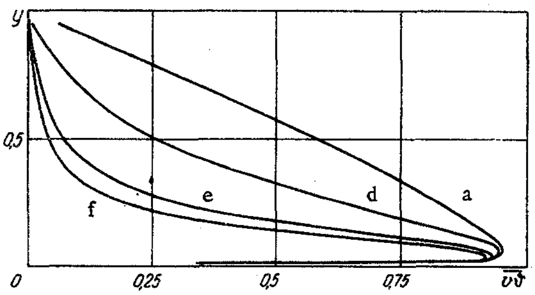
Having formulated the problem of determining the dynamic characteristics from Eqs. (1)-(8), we now turn to consider the problem of determining the thermal characteristics. We can write the equation for convective heat transfer for an incompressible fluid as

$$\frac{\partial t}{\partial \tau} + v_r \frac{\partial t}{\partial r} + \frac{1}{r} v_\varphi \frac{\partial t}{\partial \varphi} + v_z \frac{\partial t}{\partial z} = \frac{1}{r} \lambda \left\{ \frac{\partial}{\partial r}\left(\frac{\partial t}{\partial r} r\right) + \frac{\partial}{\partial \varphi}\left(\frac{\partial t}{\partial \varphi} \cdot \frac{1}{r}\right) + \frac{\partial}{\partial z}\left(\frac{\partial t}{\partial z} r\right)\right\}.$$

From this equation and the Navier–Stokes equations we can obtain† the equations for the change in the pulsation heat flows. To close this system of equations for the pulsation heat flows we use the approximation proposed in [4] for the "dissipative" terms and the terms determining the change in $\overline{u_i' t'}$ due to pressure

*The integral scale of turbulence can be defined as the solution of the corresponding differential equation [5, 6] in conjunction with the equations for the first and second moments.

†The equations are given in [4] in a Cartesian coordinate system.



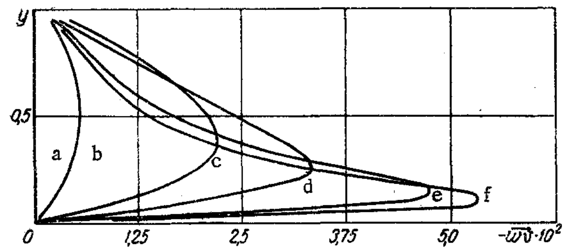


Fig. 5

Fig. 6

Fig. 5. Distribution of the transverse heat flows: a) $R_\omega = 0$; d) 10^3; e) $2.5\cdot10^3$; f) $3.5\cdot10^3$.

Fig. 6. Distribution of the azimuthal heat flows: a) $R_\omega = 0$; b) 10^2; c) $0.5\cdot10^3$; d) 10^3; e) $2.5\cdot10^3$; f) $3.5\cdot10^3$.

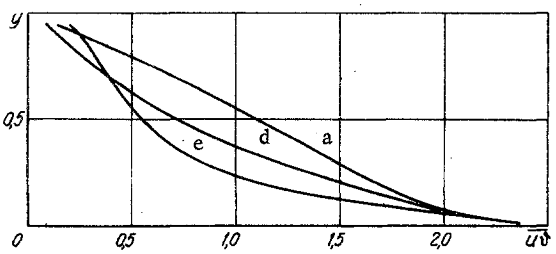


Fig. 7. Distribution of the longitudinal heat flows: a) $R_\omega = 0$; d) 10^3; e) $2.5\cdot10^3$.

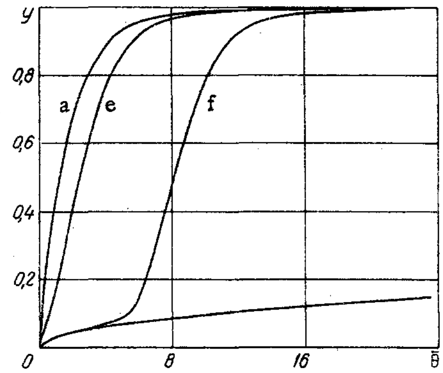


Fig. 8. Averaged temperature profile: a) $R_\omega = 0$; e) $2.5\cdot10^3$; f) $3.5\cdot10^3$.

pulsations:

$$\text{a)}\quad \Phi_i = c_{1u\vartheta}\,\lambda\,\frac{\overline{v_i't'}}{L_{u\vartheta}^2},$$

$$\text{b)}\quad P_i = -k_\vartheta\,\frac{\overline{E}^{\frac{1}{2}}}{L_{u\vartheta}}\,\overline{v_i't'}. \tag{9}$$

Here $L_{u\vartheta}$ is the integral scale defined by the spatial correlation between the pulsation velocity and the temperature.

In this paper we use the equation for the change in the pulsation heat flows (and the equations for the change in the Reynolds stresses) in the diffusionless approximation. Neglect of the turbulent diffusion is justified by the extremely small dimensions of the superlayer (where the turbulent diffusion occurs) by comparison with the radius of the pipe. In addition, we shall also neglect the viscous diffusion of the pulsation heat flows. Then, of course, we lose the possibility of calculating the pulsation thermal characteristics in the molecular sublayer (but not in the transitional region). However, the profile of the averaged temperature in the sublayer can be calculated if we assume that molecular transport dominates in comparison with molar transport.

The first integral of the averaged equation for convective heat transfer for a fully developed flow in a pipe when the heat flows in the z direction are constant, i.e., for a linear temperature distribution along the z-axis, ($t = Az + \overline{T}(r)$), has the form

$$\lambda\frac{d\overline{T}}{dr} - \overline{v_r't'} = A\frac{1}{r}\int_0^r \overline{V}_z r\,dr, \tag{10}$$

where

$$A=\frac{\partial\bar{t}}{\partial z}=-\frac{q_w}{\rho c_p}\frac{a}{\int\limits_{r=0}^{r=a}\bar{V}_z r dr};\quad q_w=-\rho c_p\lambda\left(\frac{d\bar{T}}{dr}\right)_w.$$

Recalling the approximation (9) and the condition $\partial\bar{T}/\partial z \ll \partial\bar{T}/\partial r$, and also introducing the nondimensional variables

$$\bar{\theta}=\frac{\bar{T}-T_w}{T_*};\quad \vartheta=\frac{t'}{T_*};\quad u=\frac{v'_z}{v_*};\quad v=\frac{v'_r}{v_*};$$

$$w=\frac{v'_\varphi}{v_*};\quad \bar{u}=\frac{\bar{V}_z}{v_*};\quad y=\frac{r}{a},$$

we can write the equations for the pulsation heat flows and Eq. (10) as

$$\overline{v\vartheta}\,\mathrm{R}_L+\overline{uv}\,\mathrm{R}_* l\Theta_L-\bar{u}^2\mathrm{R}_* l^2\frac{1}{\int\limits_0^1\bar{u}y dy}+c_{1u\vartheta}\frac{L_{uu}^2}{L_{u\vartheta}^2}\overline{u\vartheta}+k_\vartheta\mathrm{R}_E\frac{L_{uu}}{L_{u\vartheta}}\overline{u\vartheta}=0, \tag{11}$$

$$\overline{v^2}\,\mathrm{R}_* l\Theta_L-\overline{uv}\,\mathrm{R}_* l^2\frac{1}{\int\limits_0^1\bar{u}y dy}+c_{1u\vartheta}\frac{L_{uu}^2}{L_{u\vartheta}^2}\overline{v\vartheta}+k_\vartheta\mathrm{R}_E\frac{L_{uu}}{L_{u\vartheta}}\overline{v\vartheta}-2\mathrm{R}_{L_\omega}\overline{w\vartheta}=0 \tag{12}$$

$$\overline{vw}\,\mathrm{R}_* l\Theta_L-\overline{uw}\,\mathrm{R}_* l^2\frac{1}{\int\limits_0^1\bar{u}y dy}+c_{1u\vartheta}\frac{L_{uu}^2}{L_{u\vartheta}^2}\overline{w\vartheta}+k_\vartheta\mathrm{R}_E\frac{L_{uu}}{L_{u\vartheta}}\overline{w\vartheta}+\mathrm{R}_{L_\omega}\overline{v\vartheta}=0, \tag{13}$$

$$\frac{\Theta_L}{l}-\overline{v\vartheta}\,\mathrm{R}_*=-\mathrm{R}_*\frac{1}{y}\frac{\int\limits_0^y\bar{u}y dy}{\int\limits_0^1\bar{u}y dy}, \tag{14}$$

where $\Theta_L = L_{uu}\overline{(\partial\Theta/\partial r)}$ is the local temperature factor; $\mathrm{R}_{L_\omega} = L_{uu}^2(\omega/\nu)$ is the local Reynolds number of the rotational motion.

Thus, if we know the dynamical characteristics, from Eqs. (11)-(14) we can calculate the thermal characteristics. However, we must know $L_{u\vartheta}$. Here we assume that $L_{u\vartheta}$ is proportional to the dynamic scale L_{uu}, i.e.,

$$L_{u\vartheta}=bL_{uu}.$$

The constants $c_{1u\vartheta} = c_{1u\vartheta}/b^2$ and $k_\vartheta = k_\vartheta/b$ in the approximation equations (9), strictly speaking, must be defined experimentally. But their values can be estimated [7]. Indeed, if we consider the dynamical equations for the energy spectrum and the spectrum of the scalar substance when molecular transport dominates (small R_E) and assume the flow under consideration to be weakly anisotropic, we find that $c_{1u\vartheta} = \pi$.

In addition, if we consider the expression for the pulsation heat flows $\overline{u_z t'}$ at large R_E and compare the resulting expression with the corresponding Prandtl–Boussinesq† equation, we find that

$$k_\vartheta=k_u\frac{\sqrt{\sigma}}{2}.$$

We can also use a similar approach to determine estimates of the coefficients in the dynamic approximations [1, 2]. Then we obtain the following values for the coefficients in Eqs. (2)-(8):

$$c_{1uu}=\frac{5}{4}\pi,\quad c_{uu}=0.4\alpha,$$

$$k_u=0.96.$$

†The constant in the approximation (9b) can be determined by a different method. In particular, we can use the assumption that turbulence is locally homogeneous and isotropic for $R_E \gg 1$ and the equation for pulsation motion [8].

From the second moment balance equations (11)-(13), recalling the dynamic characteristics (Eqs. (1), (2), (4)), we can write an expression for the pulsation heat flows in the form:

$$\overline{v\vartheta} = -\frac{1}{R_*}\frac{\Theta_L}{l}\frac{R_E^2(c_{uu}R_E + c_{1uu})}{R_L^2}\frac{(k_\vartheta R_E + c_{1u\vartheta})}{(k_u R_E + c_{1uu})}\frac{(k_u R_E + c_{1uu})^2 - 4R_{L_\omega}^2}{(k_\vartheta R_E + c_{1u\vartheta})^2 + 2R_{L_\omega}^2}, \tag{15}$$

$$\overline{w\vartheta} = -\overline{v\vartheta}\frac{R_{L_\omega}}{(k_\vartheta R_E + c_{1u\vartheta})}, \tag{16}$$

$$\overline{u\vartheta} = -\overline{v\vartheta}\frac{R_L}{(k_\vartheta R_E + c_{1u\vartheta})}\left[1 + \frac{(k_u R_E + c_{1uu})}{(k_\vartheta R_E + c_{1u\vartheta})} \times \frac{(k_\vartheta R_E + c_{1u\vartheta})^2 + 2R_{L_\omega}^2}{(k_u R_E + c_{1uu})^2 - 4R_{L\omega}^2}\right]. \tag{17}$$

To determine Θ_L/l we use Eq. (14). Substituting $\overline{v\vartheta}$ from (15) in it, we obtain

$$\left[1 + \frac{R_E^2(c_{uu}R_E + c_{1u\vartheta})}{R_L^2}\frac{(k_\vartheta R_E + c_{1u\vartheta})}{(k_u R_E + c_{1uu})}\right.$$

$$\left.\times\frac{(k_u R_E + c_{1uu})^2 - 4R_{L_\omega}^2}{(k_\vartheta R_E + c_{1u\vartheta})^2 + 2R_{L_\omega}^2}\right]\frac{\Theta_L}{l} = -R_*\frac{1}{y}\frac{\int_0^y \frac{\bar{u}}{v_*}y\,dy}{\int_0^1 \frac{\bar{u}}{v_*}y\,dy}. \tag{18}$$

Finally, recalling the definition of the local temperature factor, we can find the nondimensional averaged temperature

$$\bar{\Theta} = -\int_y^1 \frac{\Theta_L}{l}\,dy. \tag{19}$$

The system of equations (15)-(19) was solved numerically on a Minsk-22 computer. The estimates of the coefficients c_{1uu}, c_{uu}, k_u, $c_{1u\vartheta}$, k_ϑ given above were used in the computations. As an approximating function for the integral scale of turbulence we used the Prandtl–Nikuradse polynomial, the change in scale due to the rotation of the fluid being neglected.

Below we give the results of calculating the dynamic and thermal characteristics for $R_* = 10^3$ and various values of the Reynolds number for the rotational motion $R_\omega = (2/l^2)R_{L_\omega}$.

As distinct from the case of a nonrotating fluid, here the longitudinal and azimuthal velocity pulsations are correlated, this correlation increasing as the rotational velocity increases (Fig. 1). As we see from Figs. 2 and 3, as R_ω increases, we observe that the intensity of the radial and azimuthal velocity pulsations decreases and at the same time there is a marked increase in the intensity of the longitudinal pulsations. The effect of the rotation on the tangential stresses $\overline{uv}$ leads to their reduction; this is so insignificant for the range of variation of R_ω under consideration that, in the scale of Fig. 3 it is practically not detectable. The reduction in the tangential turbulent stresses leads to a certain "laminarization" of the velocity profile of the averaged motion, as shown in Fig. 4.† But here we give the velocity profile curve corresponding to "laminar" conditions.

The rotation of the fluid in the pipe essentially affects the distribution of the pulsation heat flows. As we see from Fig. 5, as R_ω increases there is a marked reduction in the transverse pulsation heat flows. Figure 6 shows the distribution of the azimuthal heat flows along the radius of the pipe. When there is no rotation $\overline{w\vartheta} = 0$. As R_ω increases a sharp increase is observed in the azimuthal heat flows and, beginning with some value of R_ω, there is a changeover in $\overline{w\vartheta}$, accompanied by a displacement of the maximum of the function towards the wall.

The distribution of the longitudinal pulsation heat flows is shown in Fig. 7 for various values of R_ω. We see that as R_ω decreases, $\overline{u\vartheta}$ falls, and because the rate of increase of the longitudinal velocity pulsations exceeds the rate of diminution of the temperature pulsations, beginning with some value of R_ω, near

†Figures 1-4 are essentially additional illustrations for [3].

the axis of the pipe, there is a changeover in the distribution of the longitudinal heat flows, accompanied by an increase in this region of the correlation.

The total effect of the rotation on the pulsation characteristics appears in the "laminarization" of the averaged temperature profile. Figure 8 shows the profile of the nondimensional averaged temperature for various values of R_ω. But here we give the curve corresponding to "laminar" conditions. We see that as R_ω increases the temperature profile near the axis of the pipe is laminarized, the size of the "laminar" region increasing as R_ω increases.

NOTATION

$u = v'_z/v_*$, $v = v'_r/v_*$, $w = v'_\varphi/v_*$	are the nondimensional velocity pulsations;
$v_* = \sqrt{\tau_w/\rho}$	is the dynamical velocity;
$l = L_{uu}/a$	is the nondimensional integral scale of velocity pulsation;
$R_L = (L_{uu}^2/\nu)(dV_z/dr)$	is the local Reynolds number;
$R_E = L_{uu}/\nu \cdot \overline{E}^{(1/2)}$	is the turbulent Reynolds number;
$\overline{E} = (1/2)\sum \overline{u_i'^2}$	is the kinetic energy of pulsation;
$\overline{\varepsilon} = \overline{E}/v_*^2$	is the nondimensional kinetic energy of pulsation;
$R_* = v_* a/\nu$	is the dynamical Reynolds number;
$R_\omega = 2\omega(a^2/\nu)$	is the Reynolds number of rotational motion;
$F(k)$	is the spatial energy spectrum;
k	is the wave number;
a	is the radius of the pipe;
k_u, c_{uu}, c_{1uu}	are the coefficients in Rotta's approximation [1];
$y = r/a$	is the nondimensional coordinate;
λ	is the coefficient of thermal conductivity;
Φ_i	is the projections of the dissipative function on the coordinate axes;
P_i	is the correlation component between "pressure pulsation" and "temperature gradient pulsation";
$T_* = q_w/\rho c_p v_*$	is the characteristic temperature;
$\sigma = \nu/\lambda$	is the molecular Prandtl number;
$\overline{\Theta} = (\overline{T} - \overline{T}_0)/T_*$	is the nondimensional difference between the local temperature and the temperature on the axis of the pipe;
α	is the Heisenberg constant [9].

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